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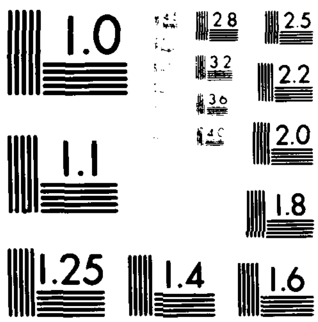

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MULTIPLE OBJECTIVE EVALUATION AND CHOICEMAKING UNDER RISK WITH PARTIAL PREFERENCE INFORMATION

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MULTIPLE OBJECTIVE EVALUATION AND CHOICEMAKING UNDER RISK WITH PARTIAL PREFERENCE INFORMATION*

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1. Introduction

The issue addressed in this paper involves the problem of selecting the k most preferred alternatives from a finite set of alternatives, where the outcome which results from any selected alternative is determined probabilistically and where the problem is characterized by multiple, noncommensurate, and conflicting objectives. Specifically, let $\Pi = \{\pi_1, \ldots, \pi_P\}$ be the finite set of all alternatives available to the decision-maker, where $k < P$. The set of all possible outcomes of each of the alternatives is $\Omega = \{\omega_1, \ldots, \omega_M\}$. Assume that N lowest level objectives and Q attributes or objectives measures have been identified. Let $x_q(\omega_m)$ be the q^{th} attribute score if the m^{th} outcome has occured, and let $x(\omega_m) = \{x_1(\omega_m), \ldots, x_Q(\omega_m)\}$. Assume for each objective n, $1 \leq n \leq N$, a value function v_n has been assessed that is isotone (monotonically nondecreasing) in preference; that is, outcome m is preferred to outcome ℓ with respect to the n^{th} objective iff (if and only if) $v_n[x(\omega_m)] > v_n[x(\omega_\ell)]$. We remark it is commonly considered desirable that each value score be associated with a single attribute which is distinct from the attributes associated with the other objectives, (Keeney and Raiffa, 1976; p. 223); that is, Q = N and $v_n[x(\omega_m)] = v_n[x_n(\omega_m)]$. We will not require this as an assumption. Let $v^m = \{v_1[x(\omega_m)], \ldots, v_N[x(\omega_m)]\} \in \mathbb{R}^N$ and $V = \{v^m, m=1, \ldots, M\}$. The finite set V is the set of all value score vectors and for our purposes is essentially equivalent to the outcome set Ω. Since more is better with respect to value scores, the relation on V considered throughout this paper is: $v'Rv$ iff $v'_n \geq v_n$ for all $n=1,\ldots, N$. Each alternative is equivalent to a probability mass function over Ω and hence equivalently over V; i.e., $\pi_p(m) = \pi_p(v^m)$ is the probability that the p^{th} alternative will result in value score vector v^m. All (subjective or objective) probabilities $\pi_p(m)$ are assumed to have been determined.

We assume that there exists a scalar-valued function $U: V \to \mathbb{R}$ such that

$$u(x) = U[v(x)]$$

where u is a multiattribute utility function and where both u and U are assumed to be unassessed. Furthermore, we assume that an objectives hierarchy has been specified and that value scores can be aggregated according to this objectives hierarchy. For example, let v_{N-1} and v_N be value scores associated with two lowest level objectives which are both associated with a single higher level objective. (The two lowest level objectives might be "increase comfort" and "decrease noise", and the higher level objective might be "improve aesthetics.") Then we assume that there exist an aggregation function $f: \mathbb{R}^2 \to \mathbb{R}^1$ and a utility function $\bar{U}$ such that

$$U(v_1,\ldots, v_N) \\ = \bar{U}[v_1,\ldots, v_{N-1}, f(v_{N-1}, v_N)].$$

These assumptions allow us to "move up" the objectives hierarchy and ultimately assess a scalar-valued value function $\bar{v}$ such that

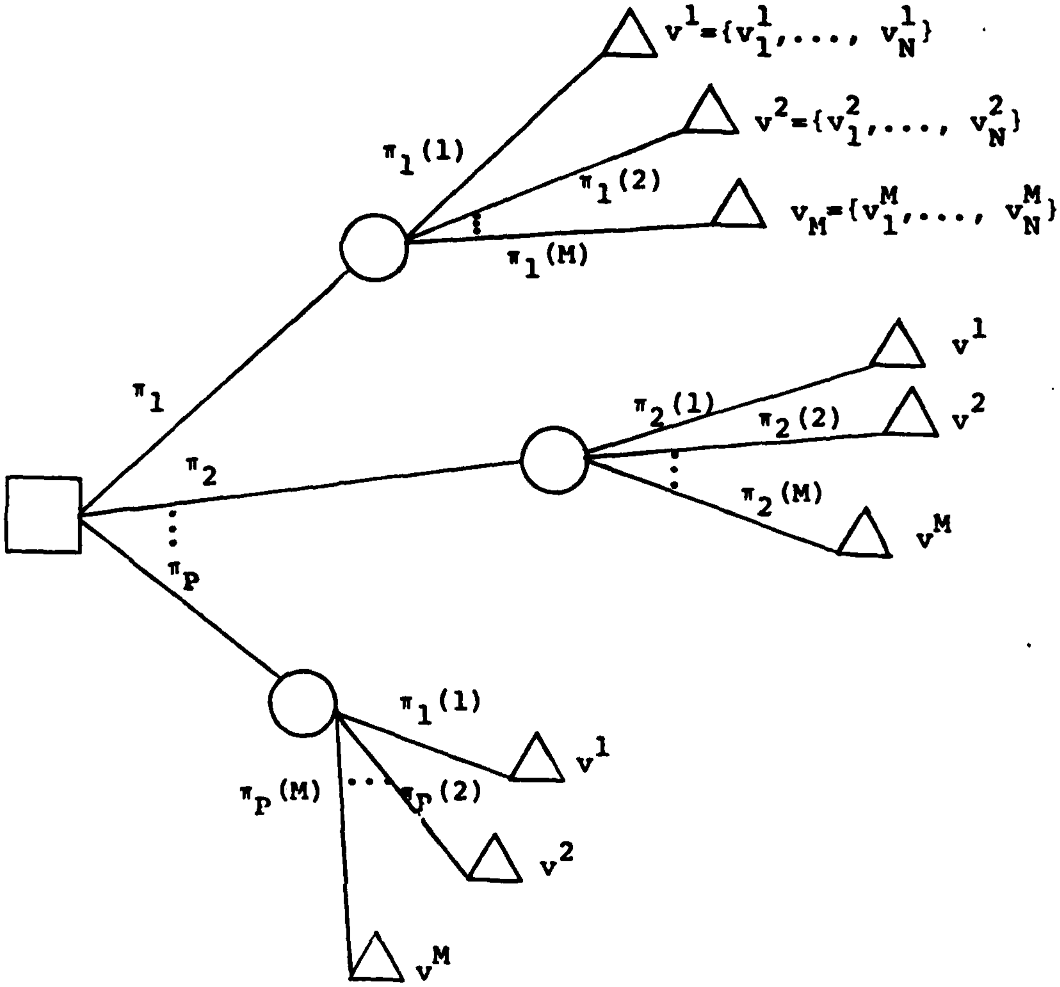


FIGURE 1: Decision Tree Representation of the Static Choicemaking Problem Under Risk with Vector-Valued Outcomes.

$u(x) = u[\bar{v}(x)]$, if we so desire.

The multiattribute utility theory (MAUT) approach for solving the problem of selecting the k most preferred alternatives is to assess a utility function $u: V \to \mathbb{R}$ which has the property that value score vector v^m is preferred to value score vector v^ℓ iff $u(v^m) > u(v^\ell)$. Alternatives can then be compared on the basis of expected utility; i.e., π' is preferred to π iff

$$\sum_{v \in V} u(v)\ \pi'(v) > \sum_{v \in V} u(v)\ \pi(v).$$

Since the expected utility generated by an alternative is a scalar, such a procedure linearly orders the alternatives and allows for the k most preferred alternatives to be easily identified.

Properly assessing the utility function, however, can be both time consuming and stressful for the decisionmaker. A choicemaking approach presented in (White and Sage, 1980), for the special case where each alternative is associated with a single outcome with probability one (i.e., the riskless case), has been developed to avoid, or reduce to the extent possible, the difficult and possibly unnecessary task of aggregating value scores. In this paper, we extend the approach presented in (White and Sage, 1980) to the case where outcomes of alternatives are described probabilistically in order to possibly reduce the time, effort, and stress often associated with MAUT utility assessment.

In order to put the results of this paper in perspective, it is useful to briefly describe the approach presented in (White and Sage, 1980). This approach is comprised of a general four step procedure for the riskless case:

1. Determine the set of alternatives that are nondominated with respect to the relation R.
2. Ask the DM (decision maker) to select the most preferred alternative from the nondominated set of alternatives.
3. If the DM can accomplish Step 2, remove the most preferred alternative from the alternative set. If the number of most preferred alternatives is less than k, go to Step 1; otherwise, stop.
4. If the DM cannot accomplish Step 2 due to the fact that the nondominated set is not small enough, then aggregate some but not necessarily all of the value scores. Typically, but not necessarily, several lower level objectives are aggregated into a single, higher level objective. Under proper assumptions, this aggregation procedure strengthens the relation R and hence will not increase the number of alternatives in the nondominated set. Go to Step 1.

The potential usefulness and behavioral relevance of the above approach are due to the facts that:

a) The most preferred alternative will always be in the nondominated set as long as the utility function is restricted to be isotone on V.

b) Alternative selection is generally easier if made from a subset of the alternative set than from the entire alternative set.

c) Selection of the k most preferred alternatives may not require explicit aggregation of all the value scores, which is required by MAUT, and may, therefore, be accomplished with relatively less time, effort, and stress.

Of course, what constitutes a "small enough" nondominated set so that Step 2 can be accomplished is highly dependent on the DM, the contingency task structure, the scope or scale of the perceptions needed to aggregate value scores, the time allotted to the task, etc. We also remark that selection of the non-dominated alternative set should sometimes be accomplished along with a sensitivity study in order to insure that the DM considers all possible candidates for the most preferred alternatives.

The aggregation procedure mentioned in Step 4 is accomplished by determining an aggregation function $f: \mathbb{R}^N \to \mathbb{R}^L$ consistent with the objectives hierarchy and the DM's desires concerning aggregation of preferences. The aggregation function typically aggregates the value scores of two or more objectives into the value score of a single, higher level objective; i.e., $L=1$. See the automobile purchasing problem in (White and Sage, 1980) for an example.

For the case where each alternative is associated with a single outcome with probability one, the alternative set and the set of value score vectors can be viewed as equivalent and hence the relation which describes preference on the value score vector set is inherited by the alternative set. Thus, for the certain case aggregating value scores to "strengthen" the relation on the value score vector set equivalently strengthens the relation on the alternative set. We will give a formal definition to a "strengthened" relation and discuss the implications of strengthening a relation beyond tending to reduce the associated nondominated set in the next section. When a one-to-one correspondence does not exist between the alternative set and the value score vector set, as is true for the problem considered here, the relations describing preference on these sets are different. It therefore does not necessarily follow that strengthening the relation on the value score vector set by properly aggregating value scores produces a stronger relation on the alternative set. If strengthening the value score vector set relation does strengthen the alternative set relation, then choice aiding for the risky case can proceed in a manner analogous to the procedure described in (White and Sage, 1980) and briefly outlined above.

In this paper we examine three relations on the alternative set which are induced by the relation on the value score vector set. The relations are associated with:

1. the expected value score vector
2. first order stochastic dominance
3. second order stochastic dominance

We show that for each of these three relations properly aggregating value scores strengthens the relation on the value score vector set which in turn induces a stronger relation on the alternative set. These three relations are considered in Sections 3, 4, and 5. Section 2 presents several preliminary results, and Section 6 summarizes the results and discusses their implications.

2. Preliminary Results and Comments

Assume ξ is a (preference-or-indifference) relation on an arbitrary set S, where $s' \xi s$ means s' is "weakly preferred" to s with respect to ξ. The strict preference relation induced by ξ is denoted by ξ_p; i.e., $s' \xi_p s$ iff $s' \xi s$ and not $s \xi s'$. We do not require ξ to be complete; therefore, two elements of S can be related by any one of four possible relationships: $s' \xi s$ and not $s \xi s'$, $s \xi s'$ and not $s' \xi s$, $s' \xi s$ and $s \xi s'$, and not $s' \xi s$ and not $s \xi s'$ (i.e., s and s' are incomparable).

Let $\xi \subseteq \xi'$ be defined as: $s' \xi s$ implies $s' \xi' s$, and define $\xi_p \subseteq \xi_p'$ similarly. Thus, if s' is preferred to s with respect to ξ, then s' is also preferred to s with respect to ξ', assuming $\xi \subseteq \xi'$. We say ξ' is <u>stronger</u> than (more properly, at least as strong as) ξ if $\xi \subseteq \xi'$ since ξ' is able to express preference between at least as many pairs of elements of S as can ξ. Several characterizations of the relationship between two relations ξ and ξ' on S, where $\xi \subseteq \xi'$, are given following preliminary definitions.

Let $\overline{S}(\xi)$ be the set of all nondominated elements in S with respect to ξ; i.e., let

$$\overline{S}(\xi) = \{s \in S: \text{ there does not exist an } s' \in S \text{ such that } s' \xi_p s\}$$

Define ξs as the set of all elements in S that are more preferred to $s \in S$ with respect to ξ; i.e., $\xi s = \{s' \in S: s' \xi s\}$. Similarly, let $s \xi = \{s' \in S: s \xi s'\}$.

LEMMA 1: (a) Let $\xi \subseteq \xi'$. Then $\xi s \subseteq \xi' s$ and $s\xi \subseteq s\xi'$ for all $s \in S$. (b) Assume $\xi_p \subseteq \xi_p'$. Then, $\overline{S}(\xi_p') \subseteq \overline{S}(\xi_p)$.

Proof: (a) The fact that $\xi s \subseteq \xi' s$ and $s\xi \subseteq s\xi'$ for all $s \in S$ follows directly from the definition of $\xi \subseteq \xi'$.

(b) To show that $\overline{S}(\xi_p') \subseteq \overline{S}(\xi_p)$, we will show that $\overline{S}(\xi_p)^c \subseteq \overline{S}(\xi_p')^c$, where c stands for complement. Let $s \in \overline{S}(\xi_p)^c$. Then, there is an $s' \in S$ such that $s' \xi_p s$ which due to $\xi_p \subseteq \xi_p'$ implies $s' \xi_p' s$. Thus, $s \in \overline{S}(\xi_p')^c$, and the proof is complete. $\square$

The first part of Lemma 1 states that for each element in a set, strengthening the relation on the set allows more elements to be identified as being weakly more preferred to the given element. Analogously, for each element in the set, strengthening a relation allows more elements to be identified as being weakly less preferred to the given element. Equivalently, for each element in the set, strengthening a relation reduces the number of elements that are incomparable.

The second part of Lemma 1 shows that if the relations are preference relations, then the nondominated set for the stronger relation is a subset of the nondominated set for the weaker relation. It has been shown (see, for example, (White and Sage, 1980)) that the most preferred element of a set is in its nondominated set if utility functions are assumed isotone. Thus, strengthening the preference relation on a set of alternatives may reduce the number of alternatives which are candidates for being the most preferred alternative. It is necessary, however, that the relation be a preference relation for the second part of Lemma 1 to hold, as the following example demonstrates.

EXAMPLE 1: We now show that if either ξ and/or ξ' are not preference relations, then $\xi \subseteq \xi'$ may not imply that $\overline{S}(\xi') \subseteq \overline{S}(\xi)$. Let $S = \{s^1, s^2\} \subseteq \mathbb{R}^2$, where $s^1 = (0,1)$ and $s^2 = (0,0)$. Define:

(i) $s' \xi\, s$ iff $s'_i \geq s_i$, $i=1,2$,

(ii) $s'\, \xi' s$ iff $s'_1 \geq s_1$.

Note that $s'\, \xi s$ implies $s'\, \xi' s$, i.e. $\xi \subseteq \xi'$, and that neither relations are preference relations, e.g. it is possible that both $s'\, \xi\, s$ and $s\, \xi\, s'$. Note also that $s^1 = \overline{S}(\xi) \subseteq \overline{S}(\xi') = S$. □

We remark that the only way a nondominated set can increase after a relation has been strengthened is if:

1. There is an element s' in the nondominated set which is strictly more preferred to an element s on the basis of the weaker relation and
2. Strengthening the relation causes the new preference structure to be indifferent between s and s'.

Such a situation is unlikely to occur in most practical settings, and therefore its possibility should be of little operational concern. One exception to this statement is when an attribute(s) is dropped from consideration in order to strengthen the relation on the alternative set and there are ties in value scores associated with the other attributes, as demonstrated by Example 1.

For the choicemaking problem under risk being considered in this paper, the objective is to use a relation R on the set of alternatives Π to describe preference between alternatives in order to determine the nondominated alternatives, or equivalently, in order to determine the alternatives that are candidates for being most preferred. If the nondominated set of alternatives is too large for choice selection, then we wish to strengthen $\mathcal{R}$. Since $\mathcal{R}$ cannot be strengthened directly, we hope that aggregating value scores will cause $\mathcal{R}$ to become more discriminating. That is, if $\mathcal{R}(R)$ is the relation on Π induced by the relation R on V, then we hope that aggregating value scores will strengthen R to R', i.e. $R \subseteq R'$, which will in turn induce a stronger relation on the alternative set, i.e. $\mathcal{R}(R) \subseteq \mathcal{R}(R')$.

Throughout the remainder of the paper, the aggregation of value scores will be described functionally as $f\colon \mathbb{R}^N \to \mathbb{R}^L$, where usually but not necessarily $L < N$. Note that f induces a partial order on X: $x' R' x$ iff $f(x') \geq f(x)$. Throughout the remainder of the paper f will be assumed to be at least isotone, which is easily shown to imply $R \subseteq R'$.

3. Expected Value Score Dominance

We now consider the case where alternatives are compared on the basis of the expected value of their value score vectors. It is shown that aggregating value scores using a linear isotone aggregation function will strengthen this relation on the alternative set. We also show that a linear, strictly isotone aggregation will strengthen the associated preference relation, thus guaranteeing no alternatives will be added to the set of nondominated alternatives as the preference relation is strengthened.

Assume that:

(i) $\pi' R_e \pi$ iff $\sum_{v \in V} v\pi'(v) \geq \sum_{v \in V} v\pi(v)$

(ii) $\pi' P_e \pi$ iff $\pi' R_e \pi$ and not $\pi R_e \pi'$.

Letting $f: \mathbb{R}^N \to \mathbb{R}^L$ be the aggregation function, we define:

(i) $\pi' R_e' \pi$ iff $\sum_{v \in V} f(v)\, \pi'(v) \geq \sum_{v \in V} f(v)\pi(v)$

(ii) $\pi' P_e' \pi$ iff $\pi' R_e' \pi$ and not $\pi R_e' \pi'$.

We refer to R_e as the expected value score relation on Π and P_e as its associated preference relation. R_e' and P_e' are the expected value score relation and preference relation, respectively, after a value score aggregation has been accomplished. Recall that a function is <u>affine</u> if it is linear plus a constant; i.e. $f(v) = Av + b$. Our main results for these relations are as follows.

LEMMA 2. (a) If f is affine and isotone, then $R_e \subseteq R_e'$.

(b) If f is affine and strictly isotone, then

$$\overline{\Pi}(P_e') \subseteq \overline{\Pi}(P_e).$$

Proof: (a) Assume $\pi' \ R_e \pi$. By the isotonicity of f,

$$f[\sum_{v \in V} v\pi'(v)] \geq f[\sum_{v \in V} v\pi(v)].$$

That f is affine then implies $\pi' \ R_e' \pi$.

(b) That $\pi' \ P_e \pi$ implies $\pi' \ P_e' \pi$ follows as above. The result then holds due to Lemma 1b. $\square$

We remark that several decision aids for the single decision node under risk case assume that the aggregation function is linear, e.g. (Kelly, 1978). It is in general necessary to assume that the aggregation function is affine, as the following example demonstrates.

EXAMPLE 2. Let

$$v^1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad v^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{ and } v^3 = \begin{bmatrix} 0.75 \\ 0.75 \end{bmatrix}$$

$$\pi'(v^1) = .2, \quad \pi'(v^2) = .2, \quad \pi'(v^3) = .6$$
$$\pi(v^1) = .3, \quad \pi(v^2) = .3, \quad \pi(v^3) = .4$$

We see that

$$\sum_{v \in V} v\pi'(v) = \begin{bmatrix} 0.65 \\ 0.65 \end{bmatrix}$$

$$\sum_{v \in V} v\pi(v) = \begin{bmatrix} 0.60 \\ 0.60 \end{bmatrix}$$

Thus, $\pi' \ R_e \pi$. Let $f(v_1, v_2) = \max \{v_1, v_2\}$, which is isotone.

Then,

$$\sum_{v \in V} f(v)\ \pi'(v) = 0.85$$

$$\sum_{v \in V} f(v)\ \pi\ (v) = 0.90$$

and hence $\pi\ R'_e\ \pi'$. Thus, $R \subseteq R'$ does not in general imply that $R_e \subseteq R'_e$ if f is isotone and nonlinear.

Consider also the case where the aggregation function is $f(v) = k_1 v_1 + k_2 v_2 + (1-k_1-k_2)\ v_1 v_2$, the multilinear case. A multilinear utility function is a necessary condition for mutual utility independence; see (Keeney and Raiffa, 1976) for details. Note that $f(v^1) = k_2$, $f(v^2) = k_1$, and $f(v^3) = 3(k_1 + k_2)/16 + 9/16$. It follows that

$$\sum_{v \in V} f(v)\pi'(v) = \frac{5}{16}(k_1 + k_2) + \frac{27}{80}$$

$$\sum_{v \in V} f(v)\pi(v) = \frac{3}{8}\ (k_1 + k_2) + \frac{9}{40}$$

Observe that if $(k_1 + k_2) \leq 9/5$, then $\pi'\ R'_e \pi$; however, when $(k_1 + k_2) \geq 9/5$, $\pi\ \ R'_e \pi'$. Thus, when the sum $k_1 + k_2$ becomes sufficiently large, $R \subseteq R'$ does not imply $R_e \subseteq R'_e$ but in fact implies that $R'_e \subseteq R_e$. $\square$

Interestingly, if the number of outcomes equals 2, i.e. $M=2$, then the affine assumption in Lemma 2 can be deleted.

COROLLARY 1: Assume $M=2$. Then, if the aggregation function $f\colon\ \mathbb{R}^N \to \mathbb{R}^L$ is isotone, $R_e \subseteq R'_e$.

Proof: Note that

$$\sum_{v \in V} v\pi'(v) \geq \sum_{v \in V} v\pi(v)$$

is equivalent to

$$v^1[\pi'(v^1)-\pi(v^1)] \geq v^2 \ [\pi'(v^1)-\pi(v^1)].$$

Assume $\pi'(v^1)-\pi(v^1)>0$, which implies $v^1 \geq v^2$. Note that $v^1 \geq v^2$ implies $f(v^1) \geq f(v^2)$, which in turn implies

$$f(v^1)[\pi'(v^1)-\pi(v^1)] \geq f(v^2)[\pi'(v^1)-\pi(v^1)]$$

which is equivalent to the desired result. The $\pi'(v^1)-\pi(v^1)=0$ case is trivial; the $\pi'(v^1)-\pi(v^1)<0$ case proceeds as above. □

The following example illustrates Lemma 2.

EXAMPLE 3. Assume there are five possible outcomes, four attributes initially under consideration, and six available alternatives, i.e. M=5, N=4, P=6. Table 1 presents assumed data, and Figure 2a displays the resulting domination digraph generated by the expected value scores listed in Table 2a. Figure 2a indicates that alternative 3 is dominated by alternatives 1, 2, and 4 and that the nondominated set of alternatives is {1, 2, 4, 5, 6}.

Assume $f(v) = Av$, where

$$A = \begin{bmatrix} .1 & .1 & .8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus, the aggregation function is linear and the trade-off weights for the three attributes 1, 2, and 3 are .1, .1, and .8, respectively. Table 2b gives the expected value scores after the aggregation and Figure 2b displays the resulting digraph. Note that the aggregation has produced a nondominated set which is not larger (it is in fact smaller) than the nondominated set before the aggregation, which is in agreement with Lemma 2a. □

	Outcome Number				
Attribute Number	1	2	3	4	5
1	10	5	5	0	5
2	10	0	0	0	0
3	3	3	10	0	3
4	5	5	5	0	10

(a) Value Scores for Each Outcome

	Outcome Number				
Alternative Number	1	2	3	4	5
1	0.6	0.1	0.2	0.1	0.0
2	0.7	0.0	0.1	0.2	0.0
3	0.3	0.1	0.0	0.4	0.2
4	0.3	0.0	0.1	0.1	0.5
5	0.1	0.1	0.0	0.1	0.7
6	0.0	0.1	0.1	0.0	0.8

(b) Probabilities for Example 3.

TABLE 1. Data for Example 3.

Alternative Numbers

		1	2	3	4	5	6
Attribute Number	1	7.5	7.5	4.5	6.0	5.0	5.0
	2	6.0	7.0	3.0	3.0	1.0	0.0
	3	4.1	3.1	1.8	3.4	2.7	3.7
	4	4.5	4.0	4.0	7.0	8.0	9.0

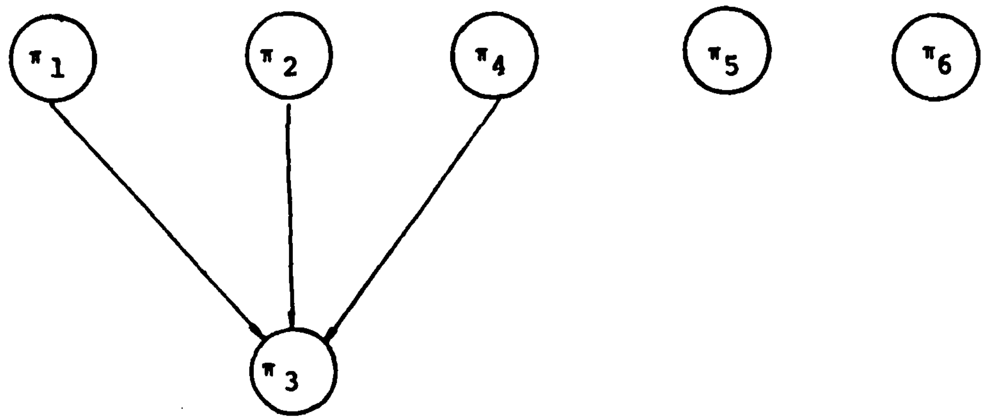
(a) Expected Value Scores Before the Aggregation

Alternative Numbers

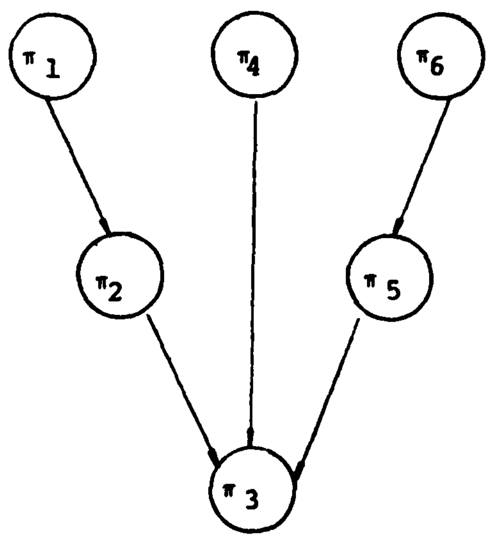
		1	2	3	4	5	6
Attribute Number	1-2-3	4.63	3.93	2.19	3.62	2.76	3.46
	4	4.50	4.00	4.00	7.00	8.00	9.00

(b) Expected Value Scores After the Aggregation

TABLE 2. Expected Value Scores for Example 3.



(a)



(b)

FIGURE 2: Domination Diagraphs Based on Expected Value Score Dominance for Example 3: (a) Before Attribute Aggregation, (b) After Attribute Aggregation.

We remark that the well-known mean-variance order (a definition is given, for example, in (Fishburn, and Vickson, 1978)) is a special case of the expected value score relation. To see this, let Q=1 and N=2, interpret $x(\omega_m)$ as the return resulting from outcome m, and define

$$v_1[x(\omega_m)] = x(\omega_m)$$

$$v_2[x(\omega_m)] = -(x(\omega_m) - \bar{x}_p)^2$$

if $\pi_p(m) \neq 0$, where $\bar{x}_p = \Sigma_m x(\omega_m)\, \pi_p(m)$. Alternatively, $v_1[x(\omega_m)] = \bar{x}_p$. In order for $v = \{v_1, v_2\}$ to be well-defined, we require that Ω and Π are modeled to have the following property: if m is such that $\pi_p(m) \neq 0$, then $\pi_q(m) = 0$ for all $q \in \Pi$, $q \neq p$. The relation R_e then becomes the mean-variance order.

4. First-Order Stochastic Dominance

In this section, we investigate multiattribute, first-order stochastic dominance as a means of defining a relation on the alternative set Π. We demonstrate that an isotone aggregation of value scores on the value score vector set V strengthens the relation on Π. An additional condition is required to prove that the preference relation on Π is strengthened.

Consider the following definitions:

(i) $\mathcal{K} = \{K \subseteq V: \ v \varepsilon K, \ v' \ \varepsilon \ V, \text{ and } v' \geq v \text{ imply } v' \ \varepsilon K\}$

(ii) $\pi' \ R_1 \pi$ iff $\pi'(K) \geq \pi(K)$ for all $K \ \varepsilon \ \mathcal{K}$

(iii) $\pi' \ P_1 \pi$ iff $\pi' \ R_1 \pi$ and not $\pi \ \ R_1 \pi'$.

Observe that (iii) is equivalent to:

(iii)' $\pi' P_1 \pi$ iff $\pi' R_1 \pi$ and there is a $K \varepsilon \mathcal{K}$ such that $\pi'(K) \neq \pi(K)$.

We refer to R_1 as the first order stochastic dominance relation on Π and P_1 as its associated preference relation. Observe that $\mathcal{K}$ represents the collection of all the so-called increasing subsets of V with respect to the usual partial order R on $\mathbb{R}^N$. As has been stated in (Fishburn and Vickson, 1978), R_1 partially orders Π; that is, R_1 is both transitive and antisymmetric.

Note that by extending the domain of the elements in Π to be the usual collection of Borel sets in $\mathbb{R}^N$, it is easy to show $\pi' R_1 \pi$ is equivalent to:

$$\int_S d\pi' \geq \int_S d\pi$$

for all increasing Borel sets S. As shown in (Lehmann, 1955) and (Fishburn and Vickson, 1978), such a statement is equivalent to the usual definition of first-order stochastic dominance:

$$\sum_{v\epsilon V} u(v)\pi'(v) \geq \sum_{v\epsilon V} u(v)\ \pi(v)$$

for all $u \epsilon U_1$, where U_1 is the collection of all isotone scalar-valued functions on $\mathbb{R}^N$. Thus, if $\pi' \ R_1 \pi$, then alternative π' can never produce a lower expected utility value than alternative π, as long as we restrict attention to isotone utility functions.

We remark that a particularly desirable aspect of the definition of the relations R_1 and P_1 is that K contains a finite number of elements (in fact, at most $\Sigma_{m=1}^{M} \binom{M}{m}$)). Therefore, a finite algorithm can easily be constructed that can test for first-order stochastic dominance.

Let f: $\mathbb{R}^N \to \mathbb{R}^L$ be the aggregation function, and let $K' = \{K \subseteq V\colon\ v\epsilon K,\ v'\epsilon V, \text{ and } f(v') \geq f(v) \text{ imply } v'\epsilon K\}$. Define R_1' and P_1' exactly as R_1 and P_1 were defined except substitute K' for K. We now present the main result of this section.

LEMMA 3. If f is isotone, then $R_1 \subseteq R_1'$.

Proof: A simple argument demonstrates that $K' \subseteq K$, which implies $R_1 \subseteq R_1'$ directly from the definition. □

Determining conditions which imply that $P_1 \subseteq P_1'$ is a more complicated task than determining conditions which imply that $R_1 \subsetneq R_1'$ and requires further definitions. Let $K_1 = \{K\epsilon K\colon K^C \ \epsilon\ K\}$, $K_2 = K_1^C$, and define K_i', i=1, 2, similarly with K' replacing K. Define $\tilde{K}(\pi',\pi) = \{K\ \epsilon\ K_2\colon\ \pi'(K) \neq \pi\ (K)\}$; observe that if $\pi'\ P_1\pi$, then $\tilde{K}(\pi',\pi) \neq \emptyset$.

LEMMA 4: Assume:

(i) f is isotone

(ii) $K_2' \cap \tilde{K}(\pi',\pi) \neq \emptyset$ for all $\pi',\pi \in \Pi$ such that $\pi' P_1 \pi$.

Then, $\overline{\Pi}(P_1') \subseteq \overline{\Pi}(P_1)$.

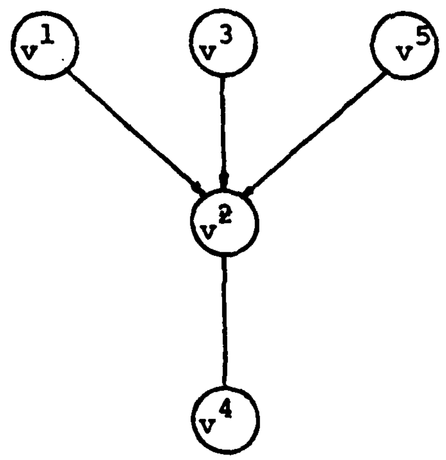
Proof: Lemma 3 implies $R_1 \subseteq R_1'$. It is easily shown that $K_2' \cap \tilde{K}(\pi', \pi) \neq \emptyset$ insures that $\pi' P_1 \pi$ implies $\pi' P_1' \pi$. Condition (ii) insures that this is true for all pairs in Π of interest and hence that $P_1 \subseteq P_2'$. The result then follows from Lemma 1b. $\square$

We remark that condition (ii) in Lemma 4 is not very meaningful operationally since K_2' becomes known only after the value score aggregation. However, as we have discussed earlier, the desired characteristic, $\overline{\Pi}(P_1') \subseteq \overline{\Pi}(P_1)$, will probably rarely be affected by whether or not condition (ii) holds.

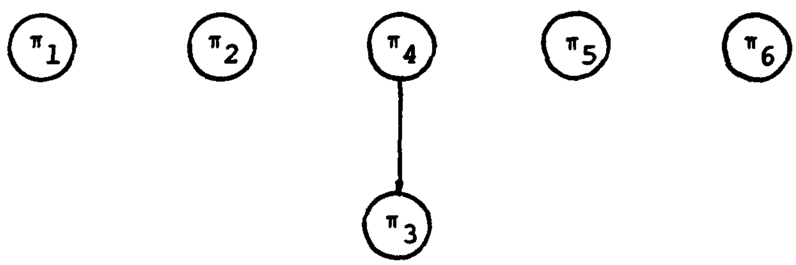
It is interesting to note that $\pi'(K)=\pi(K)$ for all $K \epsilon K_1$ if $\pi' R_1 \pi$. This fact suggests that in trying to determine if $\pi' R_1 \pi$, a simple initial check would be to see if $\pi'(K) = \pi(K)$ for all $K \epsilon K_1$. We observe that only half of the K_1 sets need to be checked since $\pi'(K) = \pi(K)$ iff $\pi'(K^c) = \pi(K^c)$.

We also remark that if all elements in the outcome set are nondominated, then $K = K_1$, and conversely. The discussion preliminary to the statement of Lemma 4 can then be used to show that $\overline{\Pi}(P_1)=\Pi$.

EXAMPLE 4. Consider the problem presented in Example 3. The data presented in Table 1a generate the digraph of elements in V in Figure 3a. The increasing sets in K associated with this digraph which are of interest are: {1}, {3}, {5}, {1,3}, {1,5}, {3,5}, {1,3,5}, and {1,2,3,5}. (We delete consideration of $\{1,2,3,4,5\} \in K$ and $\phi \in K$ since $\pi(\{1,2,3,4,5\})= 1.0$ and $\pi(\phi) = 0.0$ for all $\pi \in \Pi$.) The probabilities associated with each of these sets for each alternative are displayed in Table 3a; Figure 3b presents the related digraph of elements in Π. As in Example 3, let f be linear and assume value scores 1, 2, and 3 are traded-off with weights .1, .1, and .8, respectively. The resulting digraph of elements in V is given in Figure 4a. This digraph has the following increasing sets of interest: {3}, {5}, {3, 5}, {1, 3}, {1, 3, 5}, and {1,2,3,5}. Notice that the total number of increasing sets has been reduced by the aggregation. The digraph of elements in Π associated with the data presented in Table 3b is displayed in Figure 4b, providing more preference information with respect to the alternatives than does the digraph in Figure 3b. $\square$



(a)



(b)

FIGURE 3. Domination Digraphs Associated with First-Order Stochastic Dominance for Example 4 Before the Value Score Aggregation (a) The Value Score Digraph (b) The Alternative Digraph

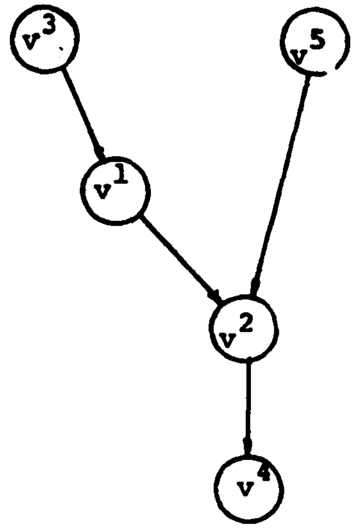
Increasing Subset	Alternative 1	2	3	4	5	6
{1}	0.6	0.7	0.3	0.3	0.1	0.0
{3}	0.2	0.1	0.0	0.1	0.0	0.1
{5}	0.0	0.0	0.2	0.5	0.7	0.8
{1, 3}	0.8	0.8	0.3	0.4	0.1	0.1
{1, 5}	0.6	0.7	0.5	0.8	0.8	0.8
{3, 5}	0.2	0.1	0.2	0.6	0.7	0.9
{1, 3, 5}	0.8	0.8	0.5	0.9	0.8	0.9
{1, 2, 3, 5}	0.9	0.8	0.6	0.9	0.9	1.0

(a)

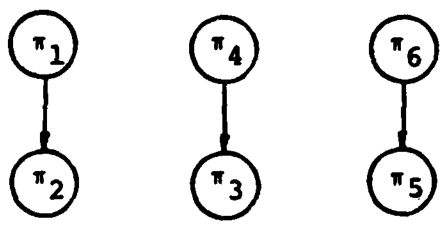
{3}	0.2	0.1	0.0	0.1	0.0	0.1
{5}	0.0	0.0	0.2	0.5	0.7	0.8
{3, 5}	0.2	0.1	0.2	0.5	0.7	0.8
{1, 3}	0.8	0.8	0.3	0.4	0.1	0.1
{1, 3, 5}	0.8	0.8	0.5	0.9	0.8	0.9
{1, 2, 3, 5}	0.9	0.8	0.6	0.9	0.9	0.0

(b)

TABLE 3: Probabilities of the Increasing Sets for Each Alternative, i.e. $P(v \epsilon K \mid \pi)$ for each $K \epsilon \mathcal{K}$ and $\pi \epsilon \Pi$ (a) Before and (b) After the Value Score Aggregation in Example 4.



(a)



(b)

FIGURE 4: Domination Digraphs Associated with First Order Stochastic Dominance for Example 4 after the Value Score Aggregation (a) The Value Score Digraph (b) The Alternative Digraph.

It is of interest to give conditions that relate the two orders $\mathcal{R}_e$ and $\mathcal{R}_1$. Let the function I_K: $V \to \{0,1\}$ be such that $I_K(v) = 1$ if $v \in K$ and $I_K(v) = 0$ if $v \notin K$. Let $I = \{I_K\}_{K \in \mathcal{K}}$. Thus, $I(v)$ is a γ-vector if $\mathcal{K}$ contains γ subsets of V which identifies the increasing sets containing v. It is easy to demonstrate that $\pi' \mathcal{R}_1' \pi$ is equivalent to

$$\Sigma_v\ I(v)\ \pi'(v) \geq \Sigma_v\ I(v)\ \pi(v)$$

It is straightforward to show that the following results hold using the same procedure as was used in the proof of Lemma 2.

LEMMA 5. (a) Assume there is an isotone, affine function α such that $\alpha[I(v)] = v$ for all $v \in V$. Then $\mathcal{R}_1 \subseteq \mathcal{R}_e$.

(b) Assume there is an isotone, affine function β such that $\beta(v) = I(v)$ for all $v \in V$. Then $\mathcal{R}_e \subseteq \mathcal{R}_1$.

We now illustrate Lemma 5 with the following example.

EXAMPLE 5. Consider the data presented in Tables 1a and 3a for Examples 3 and 4, respectively. With reference to Table 3a, note that $I(v^1)=$ col $\{1,0,0,1,1,0,1,1\}$, in that $1 \in \{1\}$, $1 \notin \{3\}$, $1 \notin \{5\}$, $1 \in \{1,3\}$, and so forth. The collection of column vectors $\{I(v^1),\ldots, I(v^5)\}$ is then:

$$[I(v^1),\ldots, I(v^5)] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

In order to illustrate Lemma 5a, we seek an isotone, affine function α such that

$$[v^1,\dots, v^5] = \alpha[I(v^1),\dots, I(v^5)]$$

where from Table 1a,

$$[v^1,\dots, v^5] = \begin{bmatrix} 10 & 5 & 5 & 0 & 5 \\ 10 & 0 & 0 & 0 & 0 \\ 3 & 3 & 10 & 0 & 3 \\ 5 & 5 & 5 & 0 & 10 \end{bmatrix}$$

Note that since $v^4 = \alpha I(v^4)$, α is required to be linear; i.e. α is a 4 x 8 matrix. We can now omit v^4 and $I(v^4)$ from further consideration. Standard algebraic procedures show that

$$\alpha = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \\ 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Clearly, α is isotone and hence from Lemma 5a, $R_1 \subsetneq R_e$, which is verified by Figures 2a and 3b. Since the aggregation function used was linear in both Examples 3 and 4, it is straightforward to show that $R_1' \subsetneq R_e'$, using Lemma 5a and the same α as above.

Straightforward algebraic manipulation shows that there exists a unique 8 x 4 matrix β such that $\beta v = I(v)$ for all $v \in V$, where

$$\beta = \begin{bmatrix} 0 & 1/10 & 0 & 0 \\ -3/35 & 3/70 & 1/7 & 0 \\ -1/5 & 1/10 & 0 & 1/5 \\ -3/35 & 1/7 & 1/7 & 0 \\ -1/5 & 1/5 & 0 & 1/5 \\ -2/7 & 1/7 & 1/7 & 1/5 \\ -2/7 & 17/70 & 1/7 & 1/5 \\ 1/5 & -1/10 & 0 & 0 \end{bmatrix}$$

Note, however, that β is not isotone and thus it does not necessarily follow from Lemma 5b that $R_e \subseteq R_1$. In fact, we observe from Figures 2a and 3b that $R_e \subseteq R_1$ does not hold. $\square$

5. Second-Order Stochastic Dominance

In this section, we examine multiattribute second-order stochastic dominance as a means of ordering the set of alternatives. Using necessary and sufficient conditions for multivariable second order stochastic dominance due to Fishburn and Vickson (1978), we show that a linear, isotone aggregation of value scores on the set V induces a strengthened relation on the alternative set Π. Here we will not define a strict preference relation on Π, noting that a stronger relation on Π will usually produce a nondominated set of alternatives after a value score aggregation which is no larger than the nondominated set of alternatives before the aggregation.

Define U_2 as the collection of all isotone, concave functions; i.e., $u \in U_1$ and $u[\lambda r + (1-\lambda)r'] \geq \lambda u(r) + (1-\lambda) u(r')$ for all $0 \leq \lambda \leq 1$ for any $r, r' \in \mathbb{R}^N$. The concavity of a utility function is the functional representation of global risk aversion, describing the preference structure of a decisionmaker who will never invest in an actuarially fair prospect. Consider the following definition:

$$\pi' R_2 \pi \text{ iff } \sum_{v \in V} u(v)\pi'(v) \geq \sum_{v \in V} u(v)\pi(v)$$

for all $u \in U_2$.

We refer to R_2 as the second order stochastic dominance relation on Π. Thus, if we know the decisionmaker is risk averse, then $\pi' R_2 \pi$ implies that alternative π' is at least as good as alternative π (with respect to the expected utility criterion). Clearly, it follows directly from the definitions that $R_1 \subseteq R_2$ and that R_2 is a partial order.

Fishburn and Vickson (1978) have presented a slightly less general version of the following necessary and sufficient conditions for $\pi' R_2 \pi$: there exists a feasible solution to the set of a linear equalities and inequalities:

(i) $d_{ij} \geq 0$ for all $i, j=1,\ldots, M$

(ii) $\sum_{j=1}^{M} d_{ij} = 1$ for all $i = 1,\ldots, M$

such that $\pi'(v^i) \neq 0$

(iii) $\pi(v^j) = \sum_{i=1}^{M} \pi'(v^i) d_{ij}$ for all $j=1,\ldots, M$

(iv) $\sum_{j=1}^{M} d_{ij} \; v_n^j \leq v_n^i$ for all $i=1,\ldots, M$

such that $\pi'(v^i) \neq 0$ and all $n=1,\ldots, N$,

where v_n^j is the n^{th} scalar entry of $v^j \in V$;

i.e. $v^j = \{v_1^j,\ldots, v_N^j\}$.

Interpretations of the solution $\{d_{ij}\}$ to the above can be found in (Fishburn and Vickson, 1978).

Let $f: \mathbb{R}^N \to \mathbb{R}^L$ be the aggregation function, and define R_2' as R_2 was defined, except replace U_2 with U_2', where

$U_1' = \{u:\ u \text{ is isotone on } f(\mathbb{R}^N)\}$

$U_2' = \{u \in U_1':\ u \text{ is concave on } f(\mathbb{R}^N)\}$.

We now present conditions on f which insure $R_2 \subseteq R_2'$.

LEMMA . Assume $f: \mathbb{R}^N \to \mathbb{R}^L$ satisfies the following conditions:

(1) $f_\ell(v^j) = \sum_{n=1}^{N} v_n^j \, b_{n\ell}$ where $b_{n\ell} \geq 0$

for all $n = 1,\ldots, N$ and $\ell = 1,\ldots, L$

(2) if $v^i \neq v^j$, then $f(v^i) \neq f(v^j)$.

Then, $R_2 \subseteq R_2'$.

Proof. We wish to show that if (iv) holds, then

$$\sum_{j=1}^{M} d_{ij} f_{\ell}(v^j) \leq f_{\ell}(v^i)$$

for all $i = 1,\ldots, M$ such that $\pi'(v^i) \neq 0$ and all $\ell = 1,\ldots, L$. (2) implies that the number of v^i such that $\pi'(v^i) \neq 0$ does not change after the aggregation is performed. (1) can be used to easily show that (iv) implies the above inequality. □

EXAMPLE 6. Again consider the problem presented in Example 3, using second-order stochastic dominance as the method of generating a partial order on Π. Figure 5 is the dominance digraph of elements in Π before the value score aggregation. Figure 2b is, coincidentally, the dominance digraph of elements in Π after the attribute aggregation. Observe that the assumption of risk aversion has caused the relationship $R_1 \subseteq R_2$ to be strict both before the aggregation (compare Figures 3b and 5) and after the aggregation (compare Figures 2b and 4b). □

It seems reasonable to pose the following computationally attractive conjecture: assume, for each $n=1,\ldots, N$, that π' stochastically dominates π in second order based on the outcome set $\{v_n^1,\ldots, v_n^M\}$; that is, assume for each $n=1,\ldots, N$, that (i) through (iii) hold and

$$\text{(iv)}' \quad \sum_{i=1}^{M} d_{ij} v_n^j \leq v_n^i$$

for all $i=1,\ldots, M$ such that $\pi'(v^i) \neq 0$. Then π' stochastically

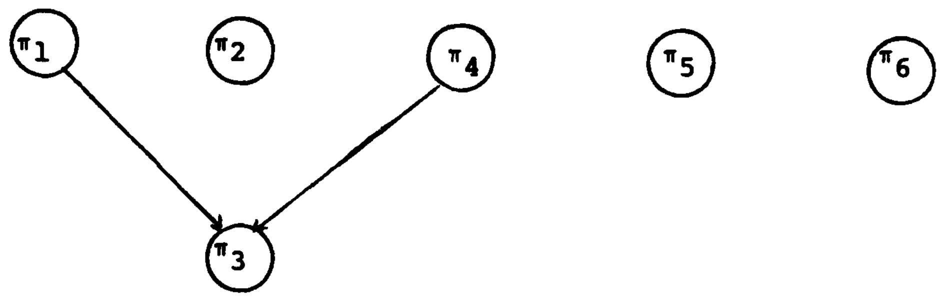


FIGURE 5. The Digraph for Example 6 Before the Value Score Aggregation

dominates π in second order based on the outcome set $\{v^1, \ldots, v^M\}$; that is, (i) through (iv) hold.

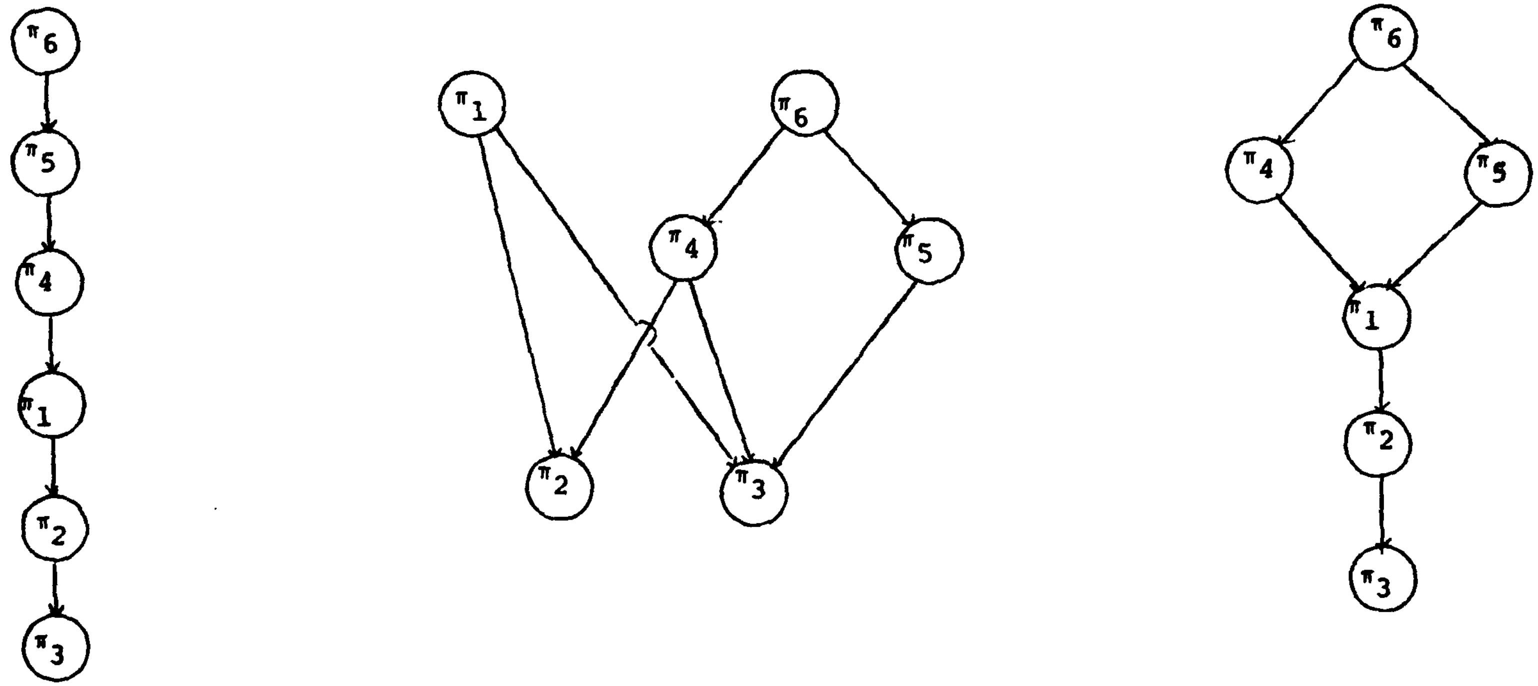
The computational attractiveness of the above conjecture is due to the fact that there exist computationally simple checks for univariate second order stochastic dominance, c.f. (Fishburn and Vickson, 1978). Unfortunately, the above conjecture is false in general since the set of all $\{d_{ij}\}$ satisfying (iv)' for n=k and (iv)' for n=k', $k \neq k'$, may be null. It has been shown that the above statement and its converse hold, c.f. Theorem 2.11 (Fishburn and Vickson, 1978), under suitable independence assumptions. These independence assumptions, however, do not imply that the sufficient conditions for $R_2 \subseteq R_2'$ presented in Lemma 6 can be relaxed.

It is important to observe that the expected value score relation and the stochastic dominance relations differ fundamentally in their interpretation of a complete value score aggregation, i.e. for the case where the aggregation function f: $\mathbb{R}^N \to \mathbb{R}^1$ (the L=1 case). In this case, R_e' is a linear order and R_1' and R_2' may only be partial orders. R_e' essentially treats f(v) as a utility function, i.e., u(x) = f[v (x)] , whereas R_1' and R_2' assume there exists a still unassessed function U which when composed with f(v) produces the desired utility function, i.e., u(x) = U[f[v(x)]]. A discussion of the usefulness of assuming the existence of such a function U, especially for modeling risk, can be found in (Bodily, 1980). An example is now presented to illuminate the fact that if there is a single attribute, presumably associated with the highest level objective and a complete value score aggregation, then the expected value score relation is linearly ordered but the stochastic dominance relations may not be.

EXAMPLE 7. Consider the problem presented in Example 3, and let the aggregation function f(v) = Av, where

$$A = [.05 \quad .05 \quad .40 \quad .50].$$

Thus, f linearly aggregates the value scores into a single, scalar value score with weights .05, .05, .40, and .50. Equivalently, f linearly aggregates the value scores for attributes 1-2-3 and 4 with a weight of .5 each, c.f. Table 2b. Figure 6 presents the relevant digraphs. Note that $R_1 \subsetneq R_e$, in agreement with Example 5, and that $R_1 \subseteq R_2$.



(a) (b) (c)

FIGURE 6. Domination Digraphs for Example 7. (a) Expected Value Score, (b) First Order Stochastic Dominance, (c) Second Order Stochastic Dominance.

6. Conclusions

In this paper, we have examined a multiattribute, single decision node decision making problem under risk. The intent has been to investigate the extension of an approach to decision aiding under certainty presented in (White and Sage, 1980) to the case where outcomes are described probabilistically. We have seen that the risky case differs fundamentally from the certain case in that the relation on the alternative set is induced, at least in the three relations studied, by the relation on the value score vector set rather than being essentially equivalent to it. This fact has lead us to examine whether or not additional preference information supplied by the decision-maker in the form of value score aggregations would be passed on to the relation on the alternative set in the form of a stronger partial ordering. Under suitable conditions for the three induced partial orders examined, this "preference transmittal" has been shown to exist.

We observe that the value score partial order is significantly less computationally demanding than the partial orders generated by the stochastic dominance notions. Use of the value score relation requires determining and comparing the weighted sum of P N-vectors. The first-order stochastic dominance relation requires comparing P vectors having up to $\Sigma_{m=1}^{M}\binom{M}{m}$ scalar elements, where each scalar element is the sum of probabilities and where $\Sigma_{m=1}^{M}\binom{M}{m}$ represents the maximum number of elements in K. The second-order stochastic dominance relation requires determining the feasibility of P(P-1) linear programs, each having up to M(M+N+2) decision variables (2M of which are artificial variables) and up to M(N+2) side constraints. Of course, these computational requirements may decrease if Lemma 5 can be used and/or if the fact that $R_1 \subseteq R_2$ is used. Clearly, the second-order stochastic dominance relation can be substantially more computationally demanding than either of the other two relations. The computational effort required to determine the dominance digraph for the first-order stochastic dominance relation and the value score relation is similar, provided that N and the number of increasing sets in K do not differ significantly and that K has been determined. Determining all of the increasing subsets (an algorithm for determing K is given in (White and Sage, 1980)), however, is significantly more computationally involved then the task of determining the domination digraph generated by the first-order stochastic dominance relation once K has been determined.

We remark that if is possible to model the mean variance order in the context of the expected value score relation without requiring Ω and Π to satisfy the property: $\pi_p(m) \neq 0$ implies $\pi_q(m) = 0$ for all $q \in \Pi$, $q \neq p$ for all m (see Section 3). To see this, let the value score vector depend on both the alternative and outcome; that is let $v_n = v_n[x(\omega_m), p]$. Assume Q=1 and N=2, and interpret $x(\omega_m)$ to be the return resulting from outcome m. We now define:

$$v_1[x(\omega_m),\ p] = x(\omega_m)$$

$$v_2[x(\omega_m),\ p] = -(x(\omega_m) - \bar{x}_p)^2.$$

Alternatively, define

$$v_1[x(\omega_m),\ p] = \bar{x}_p.$$

The resulting relation R_e is then the mean-variance order. It is straightforward to show that the results in Lemmas 2 and 3 easily generalize to the case where the value score vector can depend on both alternative and outcome. Extending the results in Sections 4 and 5 so that the value score vector can depend on the selected alternative is not so straightforward (e.g. note the obvious difficulties in defining R_1) and is a topic of future research.

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